E-Companion for "Impacts of the COVID-19 Pandemic on Grocery Retail Operations: An Analytical Model"

EC.1 Summary of Notations

Table EC.1

α_0	Base shopping rate
ψ	Retailer's health safety effort level
$\alpha(\psi)$	Adjusted-shopping rate when health safety effort level is ψ; $\alpha = \alpha(\psi) - h$
β_w	Sensitivity to store waiting time
β'_w	Sensitivity to service waiting time (delivery and curbside pickup)
β_p	Sensitivity to delivery service premium
$C(\psi)$	Safety-related cost for each shopping trip when health safety effort level is ψ
γ	Fraction of gross profit collected by the retailer from delivery service
h	Hassle of visiting the store
λ_s^i	Shopping rate of in-store customers, $i \in \{I, D, C\}$
λ_i	Online customers' shopping rate, $i \in \{d, c\}$
n	Store occupancy limit
p_c	Curbside pickup service premium
p_d	Delivery service premium
$R(\lambda)$	Gross profit per shopping session associated with the shopping rate λ
τ	Average in-store duration under the in-store and delivery modes
τ_c	Average in-store duration under the curbside pickup mode
θ_s	Fraction of in-store customers turning away in the absence of online service ($\overline{\theta}_s = 1 - \theta_s$)
θ_i	Fraction of online customers, $i \in \{d, c\}$ ($\overline{\theta}_i = 1 - \theta_i$)
w_s^i	Average in-store customers' waiting time, $i \in \{I, D, C\}$
w_i	Average service waiting time for delivery and curbside pickup, $i \in \{d, c\}$
μ_c	Curbside pickup service capacity
g	Marginal cost of hiring additional capacity for the curbside pickup service
$\underline{R}_i$	Minimum shopping amount for delivery and curbside pickup, $i \in \{c, d\}$

EC.2 Proofs

Proof of Lemma 1. Using the functional form $R(\lambda) = R_0 e^{\nu(\alpha_0 - \lambda)}$ and differentiating the function $\lambda(R(\lambda) - C)$ with respect to λ, we get the following:

$$\frac{d}{d\lambda}\left[\lambda\left(R(\lambda) - C\right)\right] = R(\lambda)(1 - \lambda\nu) - C,$$
$$\frac{d^2}{d\lambda^2}\left[\lambda\left(R(\lambda) - C\right)\right] = -R(\lambda)(2 - \lambda\nu).$$

It follows from the second derivative that $\lambda(R(\lambda)-C)$ is concave in λ for $\lambda<2/\nu$. The first-order condition for the optimal λ satisfies:

$$\lambda^* = \frac{1}{\nu} - \frac{C}{\nu R(\lambda)} < \frac{2}{\nu}.$$

Thus, the function $\lambda(R(\lambda)-C)$ increases in λ for $\lambda<\lambda^*$ and decreases in λ for $\lambda \in (\lambda^*, 2/\nu]$. For $\lambda > 2/\nu$, we have:

$$\frac{d}{d\lambda}\left[\lambda(R(\lambda)-C)\right] = R(\lambda)(1-\lambda\nu) - C < -R(\lambda) - C < 0.$$

Thus, the function $\lambda(R(\lambda)-C)$ decreases in λ for all $\lambda>\lambda^*$. This proves that the function $\lambda(R(\lambda)-C)$ is unimodal. □

Proof of Lemma 2. The analytical expression for λ_s^I (Eq. (11)) directly follows from solving Eqs. (1) and (4) for λ_s^I. The analytical expression for λ_s^D (Eq. (12)) directly follows from solving Eqs. (1) and (5) for λ_s^D. The analytical expression for λ_s^C (Eq. (13)) directly follows from solving Eqs. (1) and (6) for λ_s^C.

The decreasing nature of λ_s^I, λ_s^D, and λ_s^C can be established by showing that the first derivative of these with respect to n are negative. □

Proof of Proposition 1. We prove the proposition separately for each operating mode below.

In-store shopping mode. By differentiating Π^I (Eq. (7)) with respect to n, we obtain:

$$\frac{d\Pi^I}{dn} = \frac{d}{d\lambda}\left[(R(\lambda)-C)\lambda\right]\Big|_{\lambda=\lambda_s^I} \times \frac{d\lambda_s^I}{dn}. \tag{EC.1}$$

We first establish that $d\lambda_s^I/dn \geq 0$. The equilibrium shopping rate $\lambda_s^I>0$ in the in-store mode satisfies the following (from Eqs. (1) and (4)):

$$\lambda_s^I = \alpha - \frac{\beta_w}{n/\tau - \lambda_s^I}.$$

By differentiating λ_s^I with respect to n, we obtain:

$$\begin{aligned}\frac{d\lambda_s^I}{dn} &= \beta_w\left(\frac{1}{n/\tau-\lambda_s^I}\right)^2\left(\frac{1}{\tau} - \frac{d\lambda_s^I}{dn}\right)\\ &= \beta_w\left(w_s^I\right)^2\left(\frac{1}{\tau}-\frac{d\lambda_s^I}{dn}\right), \qquad \text{(from Eq. (4))}\end{aligned}$$

and after rearranging the terms, we obtain:

$$\frac{d\lambda_s^I}{dn} = \frac{\beta_w\left(w_s^I\right)^2}{\tau\left(1+\beta_w\left(w_s^I\right)^2\right)} > 0.$$

Therefore, the sign of $d\Pi^I/dn$ (given in Eq. (EC.1)) depends on the sign of

$$\frac{d}{d\lambda}\left[(R(\lambda)-C)\lambda\right]\Big|_{\lambda=\lambda_s^I}. \tag{EC.2}$$

Since $\lambda_s^I < \alpha$, we have $\lambda_s^I < \lambda^*$ when $\alpha \le \lambda^*$. Therefore, given that $(R(\lambda) - C)\lambda$ is unimodal (as we proved in Lemma (1)), the sign of Eq. (EC.2) is positive and $d\Pi^I/dn > 0$ (Π^I decreases with a stricter n) when $\alpha \le \lambda^*$

However, when $\alpha > \lambda^*$, the profit may increase or decrease as n decreases. In this case, the sign of Eq. (EC.2) depends on the order of λ^* and λ_s^I: If $\lambda_s^I < \lambda^*$, the sign of (EC.2) is positive, and $d\Pi^I/dn > 0$; and if $\lambda^* < \lambda_s^I$, the sign of Eq. (EC.2) is negative, and $d\Pi^I/dn < 0$. Since λ_s^I is increasing in n (as we proved in Lemma 2), for $\lambda^* < \lambda_s^I$ to hold, n needs to be above the store occupancy limit that satisfies $\lambda_s^I = \lambda^*$.

Delivery mode. By differentiating Π^D (Eq. (8)) with respect to n, we obtain:

$$\frac{d\Pi^D}{dn} = \frac{d}{d\lambda}\left[(R(\lambda) - C)\lambda\right]\bigg|_{\lambda=\lambda_s^D} \times \frac{d\lambda_s^D}{dn} \times \overline{\theta}_d. \tag{EC.3}$$

We first establish that $d\lambda_s^D/dn \ge 0$. The equilibrium shopping rate $\lambda_s^D > 0$ in the delivery mode satisfies the following (from Eqs. (1) and (5)):

$$\lambda_s^D = \alpha - \frac{\beta_w'}{n/\tau - \left(\overline{\theta}_d \lambda_s^D + \theta_d \lambda_d\right)}.$$

By differentiating λ_s^D with respect to n, we obtain:

$$\begin{aligned}
\frac{d\lambda_s^D}{dn} &= \beta_w' \left(\frac{1}{n/\tau - \left(\overline{\theta}_d \lambda_s^D + \theta_d \lambda_d\right)}\right)^2 \left(\frac{1}{\tau} - \overline{\theta}_d \frac{d\lambda_s^D}{dn}\right) \\
&= \beta_w' \left(w_s^D\right)^2 \left(\frac{1}{\tau} - \overline{\theta}_d \frac{d\lambda_s^D}{dn}\right), \qquad \text{(from Eq. (5))}
\end{aligned}$$

and after rearranging the terms, we obtain:

$$\frac{d\lambda_s^D}{dn} = \frac{\beta_w' \left(w_s^D\right)^2}{\tau\left(1 + \overline{\theta}_d \beta_w' \left(w_s^D\right)^2\right)} > 0.$$

Therefore, given that $0 < \overline{\theta}_d < 1$, the sign of $d\Pi^D/dn$ (given in Eq. (EC.3)) depends on the sign of

$$\frac{d}{d\lambda}\left[(R(\lambda) - C)\lambda\right]\bigg|_{\lambda=\lambda_s^D}. \tag{EC.4}$$

Since $\lambda_s^D < \alpha$, we have $\lambda_s^D < \lambda^*$ when $\alpha \le \lambda^*$. Therefore, given that $(R(\lambda) - C)\lambda$ is unimodal (as we proved in Lemma 1), the sign of Eq. (EC.4) is positive and $d\Pi^D/dn > 0$ (Π^D decreases with a stricter n) when $\alpha < \lambda^*$

However, when $\alpha > \lambda^*$, the profit may increase or decrease as n decreases. In this case, the sign of Eq. (EC.4) depends on the order of λ^* and λ_s^D: If $\lambda_s^D < \lambda^*$, the sign of (EC.4) is positive, and $d\Pi^D/dn > 0$; and if $\lambda^* < \lambda_s^D$, the sign of Eq. (EC.4) is negative, and $d\Pi^D/dn < 0$. Since λ_s^D is increasing in n (as we proved in Lemma 2), for $\lambda^* < \lambda_s^D$ to hold, n needs to be above the store occupancy limit that satisfies $\lambda_s^D = \lambda^*$.

Curbside pickup mode. By differentiating Π^C (Eq. (10)) with respect to n, we obtain:

$$\frac{d\Pi^C}{dn} = \frac{d}{d\lambda}\left[(R(\lambda)-C)\lambda\right]\bigg|_{\lambda=\lambda_s^C} \times \frac{d\lambda_s^C}{dn} \times \bar{\theta}_c. \tag{EC.5}$$

We first establish that $d\lambda_s^C/dn \geq 0$. The equilibrium shopping rate $\lambda_s^C > 0$ in the curbside pickup mode satisfies the following (from Eqs. (1) and (6)):

$$\lambda_s^C = \alpha - \frac{\beta_w'}{n/\tau_c - \lambda_s^C}.$$

By differentiating λ_s^C with respect to n and rearranging the terms, we obtain:

$$\frac{d\lambda_s^C}{dn} = \frac{\beta_w'\left(w^B\right)^2}{\tau_c\left(1+\beta_w'\left(w^B\right)^2\right)} > 0.$$

Therefore, given that $0 < \theta_c < 1$, the sign of $d\Pi^C/dn$ (given in Eq. (EC.5)) depends on the sign of

$$\frac{d}{d\lambda}\left[(R(\lambda)-C)\lambda\right]\bigg|_{\lambda=\lambda_s^C}. \tag{EC.6}$$

Since $\lambda_s^C < \alpha$, we have $\lambda_s^C < \lambda^*$ when $\alpha \leq \lambda^*$. Therefore, given that $(R(\lambda)-C)\lambda$ is unimodal (as we proved in Lemma 1), the sign of (EC.6) is positive, and $d\Pi^C/dn > 0$.

However, when $\alpha > \lambda^*$, the profit may increase or decrease as n decreases. In this case, the sign of (EC.6) depends on the order of λ^* and λ_s^C: If $\lambda_s^C < \lambda^*$, the sign of (EC.6) is positive, and $d\Pi^C/dn > 0$; and if $\lambda^* < \lambda_s^C$, the sign of (EC.6) is negative, and $d\Pi^C/dn < 0$. Since λ_s^C is increasing in n (as we proved in Lemma 2), for $\lambda^* < \lambda_s^C$ to hold, n needs to be above the store occupancy limit that satisfies $\lambda_s^C = \lambda^*$. $\square$

Proof of Corollary 1. From Proposition 1, we know that when $\alpha > \lambda^*$, the profit under each operating mode may increase as n becomes stricter. To prove the corollary, we prove that λ^* is decreasing in ν and ψ, which indicates that the profit may increase with stricter n for a wider range of the adjusted base shopping rates α.

We can derive the threshold value λ^*, which maximizes the function[1] $(R(\lambda)-C(\psi))\lambda$, as in Eq. (EC.7) when $R(\lambda) = R_0 e^{\nu(\alpha_0-\lambda)}$:

$$\lambda^* = \frac{1-A}{\nu}, \tag{EC.7}$$

where A follows Eq. (EC.8) in which $W(\cdot)$ is the Lambert W function:

$$A = W\left(\frac{C(\psi)}{R_0}e^{1-\alpha_0\nu}\right). \tag{EC.8}$$

[1] In the proof of Corollary 1, we use the notation $C(\psi)$ instead of C to explicitly show the dependency of C on ψ to facilitates our proof for the sensitivity of λ^* on ψ.

Sensitivity to ν. For this, we prove that $d\lambda^*/d\nu < 0$.

$$\frac{d\lambda^*}{d\nu} = \frac{1}{\nu^2}\left(A - \nu\frac{dA}{d\nu} - 1\right) \tag{EC.9}$$

For ease of exposition, we set $y = (C(\psi)/R_0)e^{1-\alpha_0\nu}$. Therefore, we write $A = W(y)$. Using the chain rule, we have:

$$\frac{dA}{d\nu} = \frac{dW(y)}{dy} \times \frac{dy}{d\nu}.$$

From the properties of the Lambert W function, we have (Weisstein 2002):

$$\frac{dW(y)}{dy} = \frac{W(y)}{y\,(1+W(y))},$$

and we get:

$$\frac{dA}{d\nu} = -\alpha_0\frac{W(y)}{1+W(y)}.$$

By plugging the above expression in Eq. (EC.9) and the fact that $1/\nu^2 > 0$, to prove $d\lambda^*/d\nu < 0$, we need to show that:

$$W(y) + \nu\alpha_0\frac{W(y)}{1+W(y)} - 1 < 0. \tag{EC.10}$$

Denoting the left-hand side of the above equality by B, at $\nu = 0$,

$$B = -1 + W(\frac{C(\psi)e}{R_0}), \tag{EC.11}$$

which is negative when $C(\psi) < R_0$. It can also be shown that B is decreasing in ν and it approaches -1 as ν goes to infinity. This proves that $B < 0$, and therefore, λ^* is decreasing in ν.

Sensitivity to ψ. For this, we prove that $d\lambda^*/d\psi < 0$, or equivalently, $dA/d\psi > 0$. For ease of exposition, we again set $y = (C(\psi)/R_0)e^{1-\alpha_0\nu}$. Therefore, we write $A = W(y)$. Using the chain rule, we have:

$$\frac{dA}{d\psi} = \frac{dW(y)}{dy} \times \frac{dy}{d\psi}.$$

Given that $C(\psi)$ is increasing in ψ, it is clear that $dy/d\psi > 0$, and therefore, it suffices to show $dW(y)/dy > 0$ to complete the proof for $dA/d\psi > 0$. From the properties of the Lambert W function, we have (Weisstein 2002):

$$\frac{dW(y)}{dy} = \frac{W(y)}{y\,(1+W(y))},$$

which is positive, given that $y > 0$. $\square$

Proof of Proposition 2. For the proof, we set $\beta_w = \beta'_w$. By subtracting λ_s^D (based on Eqs. (1) and (5)) from λ_s^I (based on Eqs. (1) and (4)), we have:

$$\begin{aligned}\lambda_s^I - \lambda_s^D &= \frac{\beta_w}{n/\tau - \left(\bar{\theta}_d \lambda_s^D + \theta_d \lambda_d\right)} - \frac{\beta_w}{n/\tau - \bar{\theta}_s \lambda_s^I} = \beta_w \frac{n/\tau - \bar{\theta}_s \lambda_s^I - n/\tau + \left(\bar{\theta}_d \lambda_s^D + \theta_d \lambda_d\right)}{\left(n/\tau - \left(\bar{\theta}_d \lambda_s^D + \theta_d \lambda_d\right)\right)\left(n/\tau - \bar{\theta}_s \lambda_s^I\right)} \\ &= \beta_w w_s^I w_s^D \left(\bar{\theta}_d \lambda_s^D + \theta_d \lambda_d - \bar{\theta}_s \lambda_s^I\right), \qquad \text{(from Eqs. (4) and (5))}, \end{aligned} \tag{EC.12}$$

Since $\beta_w w_s^I w^D > 0$, we have from Eq. (EC.12) that $\lambda_s^I \geq \lambda_s^D \iff \bar{\theta}_d \lambda_s^D + \theta_d \lambda_d \geq \bar{\theta}_s \lambda_s^I$. We have that $\bar{\theta}_s \geq \bar{\theta}_d$. Therefore, $\lambda_s^I \geq \lambda_s^D$ also implies that $\bar{\theta}_s \lambda_s^I \geq \bar{\theta}_d \lambda_s^D$. Thus, for $\bar{\theta}_d \lambda_s^D + \theta_d \lambda_d \geq \bar{\theta}_s \lambda_s^I$ to hold, it is sufficient to have $\lambda_d \geq (\bar{\theta}_s/\theta_d)\lambda_s^I$.

The condition $\lambda_d \geq (\bar{\theta}_s/\theta_d)\lambda_s^I$ simplifies to the conditions on p_d and $\underline{R}_d$ that is given in the proposition (i.e., $p_d < (\alpha_0 - (\bar{\theta}_s/\theta_d)\lambda_s^I - \beta'_w w_d)/\beta_p$ and $\underline{R}_d < R(\bar{\theta}_s \lambda_s^I/\theta_d)$). □

Proof of Proposition 3. When there is no occupancy limitation, waiting times are negligible (i.e., $w = 0$), and the in-store shopping rates $\lambda_s^I = \lambda_s^D = \alpha$. Further, when the delivery premium $p_d \in ((\alpha_0 - \alpha - \beta'_w w_d)/\beta_p, (\alpha_0 - \underline{\alpha} - \beta'_w w_d)/\beta_p)$ and the minimum shopping amount $\underline{R}_d < R(\underline{\alpha})$, we have $\underline{\alpha} < \lambda_d < \alpha$. In this range of delivery customers' shopping rate, we have the following when $\alpha > \lambda^*$:

$$(R(\lambda_d) - C)\lambda_d \geq (R(\alpha) - C)\alpha.$$

Thus, the retailer's profit $(R(\alpha) - C)\bar{\theta}_s \alpha$ under the in-store mode is smaller than its profit $(R(\alpha) - C)\bar{\theta}_d \alpha + (R(\lambda_d) - C)\theta_d \lambda_d$ under the delivery mode.

As the store occupancy limit n becomes smaller, in-store customers' respective shopping rates λ_s^I and λ_s^D under the in-store and delivery modes decrease (Lemma 2), increasing profits $(R(\lambda_s^I) - C)\bar{\theta}_s \lambda_s^I$ and $(R(\lambda_s^D) - C)\bar{\theta}_d \lambda_s^D$ from in-store customers; but, the profit from online customers $(R(\lambda_d) - C)\theta_d \lambda_d$ remains unchanged (as it does not depend on the store occupancy limit n). Thus, there may exist a finite value of store occupancy limit n at which the profit under the in-store mode exceeds that under the delivery mode. For this to happen, we must have the largest profit under in-store mode, i.e., $(R(\lambda^*) - C)\bar{\theta}_s \lambda^*$ (which is achieved at $n = n^I$ at which $\lambda_s^I = \lambda^*$) larger than $(R(\lambda_s^D) - C)\bar{\theta}_d \lambda_s^D + (R(\lambda_d) - C)\theta_d \lambda_d$. In other words,

$$\bar{\theta}_s \geq \frac{(R(\lambda_s^D) - C)\lambda_s^D}{(R(\lambda^*) - C)\lambda^*}\bar{\theta}_d + \frac{(R(\lambda_d) - C)\lambda_d}{(R(\lambda^*) - C)\lambda^*}\theta_d.$$

This condition is met if we have:

$$\bar{\theta}_s \geq \frac{(R(\lambda_s^D) - C)\lambda_s^D}{(R(\lambda^*) - C)\lambda^*}\bar{\theta}_d + \theta_d.$$

Thus, of this condition is met, under sufficiently strict store occupancy limit, retailer earns lower profit under delivery mode than under in-store mode. □

Proof of Proposition 4. For the proof, we set $\beta_w = \beta'_w$. We prove that $\lambda_s^I > \lambda_s^C \iff \frac{n}{\tau} - \frac{n}{\tau_c} > \theta_c \lambda_s^C$. By subtracting λ_s^C from λ_s^I, we have:

$$\begin{aligned}\lambda_s^I - \lambda_s^C &= \frac{\beta_w}{n/\tau_c - \bar{\theta}_c\lambda_s^C} - \frac{\beta_w}{n/\tau - \bar{\theta}_s\lambda_s^I} = \beta_w \frac{n/\tau - \bar{\theta}_s\lambda_s^I - n/\tau_c + \bar{\theta}_c\lambda_s^C}{\left(n/\tau_c - \bar{\theta}_c\lambda_s^C\right)\left(n/\tau - \bar{\theta}_s\lambda_s^I\right)} \\ &= \beta_w w_s^I w^C \left(n/\tau - \bar{\theta}_s\lambda_s^I - n/\tau_c + \bar{\theta}_c\lambda_s^C\right), \qquad \text{(from Eqs. (4) and (6))} \end{aligned} \tag{EC.13}$$

Since β_w, w^I, and w_s^C are all positive, it follows from Eq. (EC.13) that $\lambda_s^I - \lambda_s^C$ and $\frac{n}{\tau} - \frac{n}{\tau_c} - \theta_c\lambda_s^C$ have the same sign; i.e.,

$$\lambda_s^I > \lambda_s^C \iff \frac{n}{\tau} - \bar{\theta}_s\lambda_s^I - \frac{n}{\tau_c} + \bar{\theta}_c\lambda_s^C > 0 \implies \frac{n}{\tau} - \frac{n}{\tau_c} > \left(\bar{\theta}_s - \bar{\theta}_c\right)\lambda_s^C. \tag{EC.14}$$

□

Proof of Proposition 5. We first prove the proposition for the comparison between the in-store and curbside pickup modes, and then we prove the proposition for the comparison between the delivery and curbside pickup modes.

In-store vs. curbside pickup. The expressions for the expected profits are as follow:

$$\begin{aligned}\Pi^I &= \left(R(\lambda_s^I) - C\right)\bar{\theta}_s\lambda_s^I \\ \Pi^C &= \left(R(\lambda_s^C) - C\right)\bar{\theta}_c\lambda_s^C + \left(R(\lambda_c) + p_c\right)\theta_c\lambda_c - (1-x)g\mu_c,\end{aligned}$$

Taking the difference between the two results in:

$$\Pi^C - \Pi^I = \left(R(\lambda_s^C) - C\right)\bar{\theta}_c\lambda_s^C + \left(R(\lambda_c) + p_c\right)\theta_c\lambda_c - (1-x)g\mu_c - \left(R(\lambda_s^I) - C\right)\bar{\theta}_s\lambda_s^I.$$

We substitute the term $\left(R(\lambda_s^I) - C\right)\bar{\theta}_s\lambda_s^I$ in the above expression with $\left(R(\lambda_s^I) - C\right)\lambda_s^I$, and after rearranging the terms derive the following inequality (since $\bar{\theta}_s < 1$):

$$\Pi^C - \Pi^I > \left[\left(R(\lambda_s^C) - C\right)\lambda_s^C - \left(R(\lambda_s^I) - C\right)\lambda_s^I\right] + \theta_c\left[\left(R(\lambda_c) + p_c\right)\lambda_c - \left(R(\lambda_s^C) - C\right)\lambda_s^C\right] - (1-x)g\mu_c.$$

The retailer can set the curbside premium p_c and the minimum required shopping amount $\underline{R}_c$ such that the resulting shopping rate λ_c satisfies $\left(R(\lambda_c) + p_c\right)\lambda_c \geq R(\lambda_s^C)\lambda_s^C$ (in the worst case, this can be achieved by setting p_c or $\underline{R}_c$ such that $\lambda_c = \arg\max\{R(\lambda)\lambda\}$). This results in:

$$\Pi^C - \Pi^I > \left[\left(R(\lambda_s^C) - C\right)\lambda_s^C - \left(R(\lambda_s^I) - C\right)\lambda_s^I\right] + C\theta_c\lambda_s^C - (1-x)g\mu_c. \tag{EC.15}$$

The profit function $(R(\lambda) - C)\lambda$ is unimodal (Lemma 1). Therefore, when $\lambda_s^I \neq \lambda^*$, there are two λ values for which $\lambda = \Pi^{-1}(\lambda_s^I)$ (one of these λ values is λ_s^I and the other one is either greater or smaller than λ_s^I depending on whether $\lambda_s^I < \lambda^*$ or $\lambda_s^I > \lambda^*$, respectively).[2] Let λ_l and λ_u denote the smaller and greater values of λ for which $\lambda = \Pi^{-1}(\lambda_s^I)$. As long as $\lambda_s^C \in [\lambda_l, \lambda_u]$, we have that $\left[\left(R(\lambda_s^C) - C\right)\lambda_s^C - \left(R(\lambda_s^I) - C\right)\lambda_s^I\right] \geq 0$. Under this condition, we have $\Pi^C \geq \Pi^I$ if $C\theta_c\lambda_s^C - (1-x)g\mu_c \geq 0$, which results in $C \geq ((1-x)g\mu_c)/(\lambda_s^C\theta_c)$, or equivalently, $\psi \geq C^{-1}((1-x)g\mu_c/(\lambda_s^C\theta_c))$.

[2] When $\lambda_s^I = \lambda^*$, the only λ that satisfies $\lambda = \Pi^{-1}(\lambda_s^I)$ is λ_s^I. The following discussion applies to this case as well.

Delivery vs. curbside pickup. We consider the comparison between the delivery and curbside pickup modes for $\theta_d = \theta_c = \theta$. The expressions for the expected profits are as follow:

$$\Pi^D = (R(\lambda_s^D) - C)\bar{\theta}\lambda_s^D + (\gamma R(\lambda_d) - C)\theta\lambda_d$$
$$\Pi^C = \left(R(\lambda_s^C) - C\right)\bar{\theta}\lambda_s^C + (R(\lambda_c) + p_c)\,\theta\lambda_c - (1-x)g\mu_c,$$

Taking the difference between the two results in:

$$\Pi^C - \Pi^D = \left[(R(\lambda_s^C) - C)\lambda_s^C - (R(\lambda_s^D) - C)\lambda_s^D\right]\bar{\theta} + \left[(R(\lambda_c) + p_c)\,\lambda_c - (\gamma R(\lambda_d) - C)\lambda_d\right]\theta - (1-x)g\mu_c.$$

The retailer can set the curbside premium p_c and the minimum required shopping amount $\underline{R}_c$ such that the resulting shopping rate λ_c satisfies $(R(\lambda_c) + p_c)\,\lambda_c \geq \gamma R(\lambda_d)\lambda_d$. This results in:

$$\Pi^C - \Pi^D \geq \left[(R(\lambda_s^C) - C)\lambda_s^C - (R(\lambda_s^D) - C)\lambda_s^D\right]\bar{\theta} + C\theta\lambda_d - (1-x)g\mu_c.$$

The profit function $(R(\lambda) - C)\lambda$ is unimodal (Lemma 1). Therefore, when $\lambda_s^D \neq \lambda^*$, there are two λ values for which $\lambda = \Pi^{-1}(\lambda_s^D)$ (one of these λ values is λ_s^D and the other one is either greater or smaller than λ_s^D depending on whether $\lambda_s^D < \lambda^*$ or $\lambda_s^D > \lambda^*$, respectively).[3] Let λ_l' and λ_u' denote the smaller and greater values of λ for which $\lambda = \Pi^{-1}(\lambda_s^D)$. As long as $\lambda_s^C \in [\lambda_l', \lambda_u']$, we have that $[(R(\lambda_s^C) - C)\,\lambda_s^C - (R(\lambda_s^D) - C)\,\lambda_s^I] \geq 0$. Under this condition, we have $\Pi^C \geq \Pi^D$ if $C\theta\lambda_d - (1-x)g\mu_c \geq 0$, which results in $C \geq ((1-x)g\mu_c)/(\lambda_d\theta)$, or equivalently, $\psi \geq C^{-1}((1-x)g\mu_c/(\lambda_d\theta))$. $\square$

EC.3 Parameter Values Used in the Numerical Work

Across all plots, we use $\alpha_0 = 2$, $\alpha_{min} = 1$, $\eta = 1$, $R_0 = 10$, $\nu_R = 1$, $\tau = 10$, $\beta_w = 1$, $h = 0.1$, $C(\psi) = 0.1 \times \psi$ and $\theta_s = 0$. For the delivery mode we use $\beta_w' = 0.1 \times \beta_w$, $\beta_p = 1$, $p_d = 0.5$, $\underline{R}_d = 10$, $\theta_d = 0.2$, and $\gamma = 0.95$. For the curbside pickup mode, we use $w_c = 1$, $p_c = 0.25$, $\underline{R}_c = 10$, $g = 1$, $f(x) = (1 + x^2)\tau$ with $x = 0.5$ and $\theta_c = 0.2$.

- Fig. 1: ψ is varied between 0 and 5, and n is varied between 0 and 50.
- Fig. 2: Across all plots, ψ is varied between 0 and 5, and n is varied between 0 and 50. Additionally: In Fig. 2a, θ_d takes values 0.1, 0.2, and 0.3. In Fig. 2b, $\underline{R}_d$ is varied to achieve $\lambda_d = 0.6 \times \alpha_0$, $\lambda_d = 0.7 \times \alpha_0$, and $\lambda_d = 0.8 \times \alpha_0$. In Fig. 2c, θ_s takes values 0, 0.05, and 0.1. In Fig. 2d, τ_d (average shopping time by delivery customers) takes values $1.00 \times \tau$, $0.85 \times \tau$, and $0.7 \times \tau$, and a M/M/1 model with two customer classes is used for carrying out calculations.
- Fig. 3: Across all plots ψ is varied between 0 and 5, and n is varied between 0 and 50. Additionally: In Fig. 3a, θ_c takes values 0.1, 0.2, and 0.3. In Fig. 3b, $\underline{R}_c$ is varied to achieve $\lambda_c = 0.6 \times \alpha_0$, $\lambda_c = 0.7 \times \alpha_0$, and $\lambda_c = 0.8 \times \alpha_0$. In Fig. 3c, we change the formula $f(x) = (1 + x^2)\tau$, to allow $\tau_c = 1.25 \times \tau$, $\tau_c = 1.50 \times \tau$, and $\tau_c = 1.75 \times \tau$. In Fig. 3d, θ_s takes values 0, 0.05, and 0.1.

[3] When $\lambda_s^D = \lambda^*$, the only λ that satisfies $\lambda = \Pi^{-1}(\lambda_s^D)$ is λ_s^D. The following discussion applies to this case as well.

- Fig. 4: Across all plots ψ is varied between 0 and 5, and n is varied between 0 and 50. Additionally: In Fig. 4a, $\theta_d^{'} = \theta_c^{'} = 0.1$. In Fig. 4b, three different values of $\theta_c^{'}$ and $\theta_d^{'}$ are chosen, keeping their sum $\theta_c^{'} + \theta_d^{'} = 0.2$. In Fig. 4c, $\theta_d^{'} = \theta_c^{'} = 0.05$, $\theta_d^{'} = \theta_c^{'} = 0.1$, and $\theta_d^{'} = \theta_c^{'} = 0.15$.
- Fig. 5: τ is replaced with $\mathcal{T}(\lambda_s) = \tau_0 + \kappa \times e^{\nu'(\alpha_0 - \lambda_s)}$, and three different combinations of κ and τ_0 values are chosen.

EC.4 Combined Mode (Adding Both Online Delivery and Curbside Pickup)

In this section, we consider a model in which the retailer employs both delivery and curbside pickup services. We continue considering that the delivery service is offered in partnership with a third-party delivery firm, whereas the curbside pickup service is offered using the retailer's resources. Let θ'_d and θ'_c denote the fractions of delivery and curbside pickup customers under the combined model; then $1 - \theta'_d - \theta'_c$ represents the fraction of in-store customers.

Using the structure for the shopping rates under the delivery mode and the curbside pickup mode, we can express the shopping frequency of delivery and curbside pickup customers under the combined mode as follows:

$$\lambda_d = \min\left\{ \left(\alpha_0 - \beta_p p_d - \beta'_w w_d\right)^+, R^{-1}(\underline{R}_d) \right\},$$
$$\lambda_c = \min\left\{ \left(\alpha_0 - h - \beta_p p_c - \beta'_w w_c\right)^+, R^{-1}(\underline{R}_c) \right\}.$$

As the main models, the equilibrium shopping rate for in-store customers is determined by the following two equations where w_s is the store waiting time experience by in-store customers when the store occupancy limit is n:

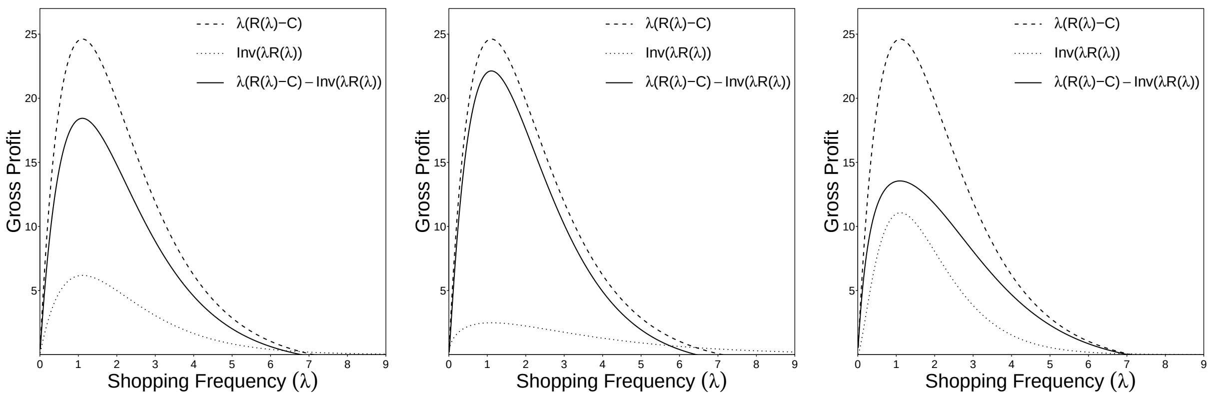
$$\lambda_s = \left(\alpha - \beta_w w_s\right)^+,$$
$$w_s = \frac{1}{n/\tau_c - ((1 - \theta_d - \theta_c)\lambda_s + \theta_d \lambda_d)}.$$

We can express the profit function under the combined mode as follows, where $\mu_c = \lambda_c + 1/w_c$ is the required capacity for the curbside pickup service to achieve the service waiting time of w_c and x is the portion of that capacity met using the current resources:

$$\Pi^{CD} = (1 - \theta_d - \theta_c)\left(R(\lambda_s^D) - C(\psi)\right)\lambda_s^D + \theta_d\left(\gamma R(\lambda_d) - C(\psi)\right)\lambda_d$$
$$+ \theta_c\left(R(\lambda_c) + p_c\right)\lambda_c - (1 - x)\, g\mu_c.$$

EC.5 Inclusion of Inventory Related Costs to the Model

This section briefly discusses how the inclusion of inventory-related costs may affect our results. As noted in §4 (the discussion accompanying Lemma 1), our results rely on the function $\lambda\left(R(\lambda) - C\right)$ being unimodal in shopping frequency λ (in this section, we refer to this function as *profit rate function*). In what follows, we illustrate that in most reasonable circumstances, including inventory



(a) Linear Inventory Cost **(b)** Concave Inventory Cost **(c)** Convex Inventory Cost

Figure EC.1 Gross profit function with the inclusion of inventory cost.

costs (i.e., the costs associated with carrying inventory and placing orders) does not change the unimodal nature of the profit rate function. Define inventory cost function $Inv(r)$ as the average inventory costs per unit time incurred in supporting the sales per unit time that leads to gross profit r per unit time. Then the inclusion of inventory cost changes the gross profit function as follows:

$$\pi^{'}(\lambda) = \lambda\left(R(\lambda) - C\right) - Inv(\lambda R(\lambda)).$$

It is reasonable to assume that the retailer's rate of earning gross profit, namely $\lambda R(\lambda)$, is linear in the sales volume, and since inventory cost increases with sales, it follows that $Inv(r)$ is an increasing function of r. There can be three possible cases for the shape of this function:

• First, inventory cost could be linear in r. This would be the case when the retailer cannot re-optimize its inventory policies to changes in product volumes, resulting in linearly increasing inventory costs. Given the temporary nature of the pandemic, this is a likely scenario. In this case, there is a linear shift in the function $\lambda(R(\lambda) - C)$, which preserves the unimodal nature of the function and the overall qualitative nature of our results. We depict this in Fig. EC.1(a), where $Inv(r) = 0.25 \times r$ (other parameter values and functional forms are the same as described in §EC.3).

• The second possibility is that $Inv(r)$ is concave in r. In other words, the inventory cost is proportionally smaller at larger volumes, i.e., the retailer can exploit economies of scale in inventory costs by re-optimizing its inventory decisions. In this case, since the inventory cost increases by a small amount compared to $\lambda R(\lambda)$, the inventory cost adjusted profit rate function remains unimodal. We depict this in Fig. EC.1(b), where $Inv(r) = 0.25 \times \sqrt{r}$.[4]

[4] This is similar to the classic economic order quantity (i.e., EOQ) formulation, in which the optimal inventory cost is proportional to the square root of flow rate.

- Finally, the third possibility is that $Inv(r)$ is convex in r. This corresponds to situations in which the retailer struggles to keep up with the demand and must incur higher inventory costs to ensure sufficient supply (for example, the retailer resorts to more expensive freight/supplier for delivering products). Even in these cases, as long as the magnitude of inventory cost is relatively small, we expect the unimodal nature of function $\pi^{'}(r)$ is preserved, as shown in Fig. EC.1(c), where $Inv(r) = 0.1 \times r^{1.5}$. However, it is indeed possible that the magnitude of cost is not small, and the resulting profit rate function is no longer unimodal. In such cases, the retailer's shopping mode decision must account for the implication of the bulk-shopping behavior on inventory costs.

References

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